Review of Xylophone

Subtask 1

In this subtask, we can send N^2 queries. So we can send queries for all the intervals.

First, we consider an interval for which the value at one of the ends of the interval is the maximum or minimum. We call such an interval a **good interval**.

Since the situation is symmetric, in the following, we only consider an interval $A[M] \sim A[M+K-1]$ whose leftmost value $A[M]$ is the minimum of the interval.

The return value $\text{Query}(M, M+K-1)$ is the difference of the maximum and the minimum of the interval. We denote it by D.

If we ask $\text{Query}(M, M+K-1)$, $\text{Query}(M, M+K-2), \ldots$ in this order, then the return values are non-increasing.

For the smallest integer i with $\text{Query}(M, M+K-i) \neq D$, the value $A[M+K-i+1]$ is the maximum of the interval, and we know this value.

Then, both of the interval $A[M] \sim A[M+K-i]$ and the interval $A[M+K-i+1] \sim A[M+K-1]$ are good intervals. Repeating this process, we can calculate all the values in a good interval.

It is enough to divide the initial sequence into two good intervals. If we ask $\text{Query}(1, N)$, $\text{Query}(1, N-1), \ldots$ in this order, then, for the smallest integer i with $\text{Query}(1, i) \neq N-1$, the integer $A[i+1]$ is the maximum. Then, both of the interval $A[1] \sim A[i+1]$ and the interval $A[i+1] \sim A[N]$ are good intervals.

Subtask 2

In the solution of Subtask 1, we can do binary search instead of ``asking in this order.''

Then we can calculate one value by $\log N$ queries.

Therefore, in total, we can solve this task by $N \log N$ queries.

Subtask 3 (Full Score)

We send queries for all the adjacent pairs ($N-1$ pairs) and all the adjacent triples ($N-2$ triples).

For every triple $A[i], A[i+1], A[i+2]$,

- if $\mathrm{Query}(i, i+2) = \mathrm{Query}(i, i+1) + \mathrm{Query}(i+1, i+2)$ holds, then the relation between $A[i]$ and $A[i+1]$ (increasing/decreasing) and the relation between $A[i+1]$ and $A[i+2]$ (increasing/decreasing) are the same, and
- if $\mathrm{Query}(i, i+2) \neq \mathrm{Query}(i, i+1) + \mathrm{Query}(i+1, i+2)$ holds, then then the relation between $A[i]$ and $A[i+1]$ (increasing/decreasing) and the relation between $A[i+1]$ and $A[i+2]$ (increasing/decreasing) are different.

We know the difference of two adjacent numbers since we already asked for them.

Adding differences appropriately (according to increasing/decreasing relation), we know all the relative values of numbers.

Finally, looking at the maximum and the minimum, we change the signs if necessary, and add adequate constants.